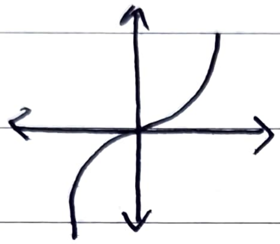
★ we'll be working in two dimention.

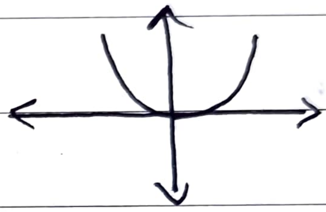
★

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$ quadratic formula

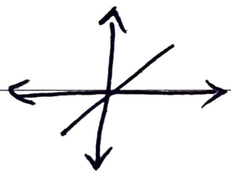
★ $y = x^3$



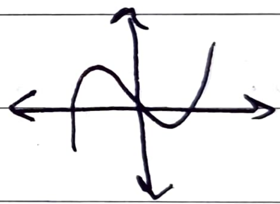
$y = x^2 + x$



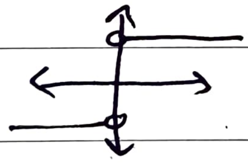
$y = x$



$y = x(x^2-1)$

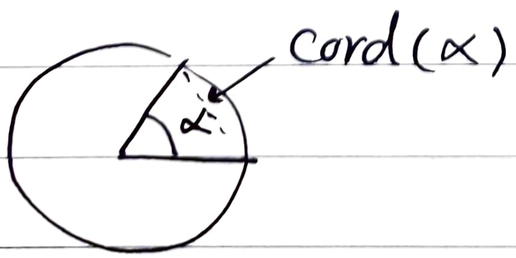


$f(x) = \frac{x}{|x|}$



The distance from x

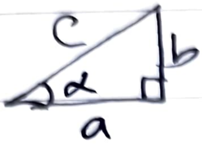
$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x \leq 0 \end{cases}$$



This was made for studying [redacted] astronomy.

No. Date.

☆ Trigonometric functions:

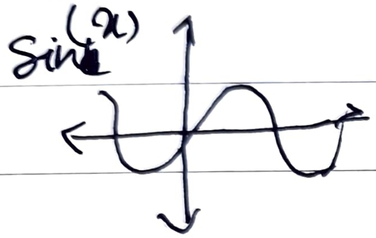


$$\cos(\alpha) = \frac{a}{c}$$

$$\sin(\alpha) = \frac{b}{c}$$

$$\tan(\alpha) = \frac{b}{a}$$

$$\sec(\alpha) = \frac{c}{a}$$



☆ Rational Functions:

$$f(x) = \frac{x^2+3x+14}{x^3+3x+14} = \frac{\text{Polynomial}}{\text{Polynomial}}$$

☆ Logarithmic & exponential functions:

$a>0$ $\quad a^x$ makes sense for all x.

$$\log_a(x) = y \iff a^y = x$$

$a = e$ $\quad e^x \quad \log_e(x) = \ln(x)$

natural base

$a = e^{\ln(a)}$

$a^x = (e^{\ln(a)})^x \Rightarrow e^{x\ln(a)} \Rightarrow (e^x)^{\ln(a)}$

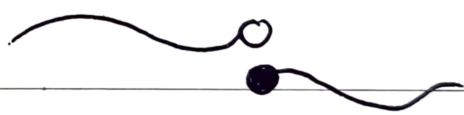
$\log_a(x) = \dfrac{\ln(x)}{\ln(a)}$

★ what does it mean when a function is continuous? $y = f(x)$

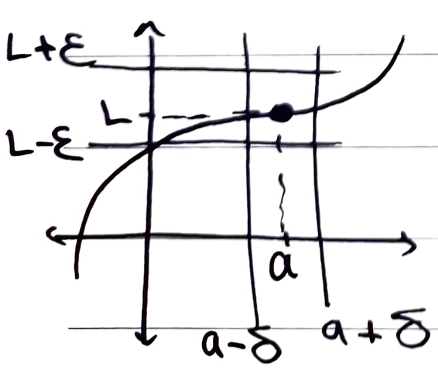
- it means there're no gaps in the graph

"Removable" discontinuity.

"jump" discontinuity



$\lim_{x \to a} f(x) = L$ means: for any $\varepsilon > 0$, there's a $\delta > 0$, so that if $|x - a| < \delta$ (is $a - d < x < a + d$) then $|f(x) - L) < \varepsilon$ is $(L - \varepsilon < f(x) < L + \varepsilon)$



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$\lim_{x \to 0} (3x+12) = 3(0)+12 = 12$

- given $\varepsilon > 0$, we need to find $\delta > 0$, so that if $|x-0| < d$ then $|(3x+12)-12| < \varepsilon$.

- suppose we are given an $\varepsilon > 0$. how do we manufacture the corresponding δ?

- we reverse-engineer the δ:

$|(3x+12)-12| < \varepsilon$

$|3x| < \varepsilon \Rightarrow |x| < \frac{\varepsilon}{3} \Rightarrow |x-0| < \frac{\varepsilon}{3}$

$|x-0| < \delta$ so $\delta = \frac{\varepsilon}{3}$

- every step is reversable so we can backtrack the process to get:

$|(3x+12)-12| < \varepsilon$

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1° (Sum Rule) $\lim_{x \to a} [f(x) + g(x)] \Rightarrow$

$[\lim_{x \to a} f(x)] + [\lim_{x \to a} g(x)]$

* all three limits must be defined.

2° (composition Rule)

suppose $\lim_{x \to a} f(x) = L$ & $\lim_{\beta \to b} g(x) = a$,

then $\lim_{\beta \to b} f(g(x)) = \lim_{\beta \to b} (f \circ g)(x) = L$

3° (multiplication Rule by contrast)

$\lim_{x \to a} c\, f(x) = cL$ if $\lim_{x \to a} f(x) = L$

4° (product rule) $\lim_{x \to a} f(x) = L$ & $\lim_{x \to a} g(x) = M$

$\Rightarrow \lim_{x \to a} f(x) g(x) = LM$

5° (Quotient Rule) $\lim_{x \to a} f(x) = L$ & $\lim_{x \to a} g(x) = M$

$\Rightarrow \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{L}{M}$

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★ Special limits to memorize :

1. $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$

2. $\lim_{x \to 0} \frac{e^x - 1}{x} = +1$

6° (squeeze thm) $\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$ &

$f(x) \leq h(x) \leq g(x)$ ★for x near a

then: $\lim_{x \to a} h(x) = L$ too.